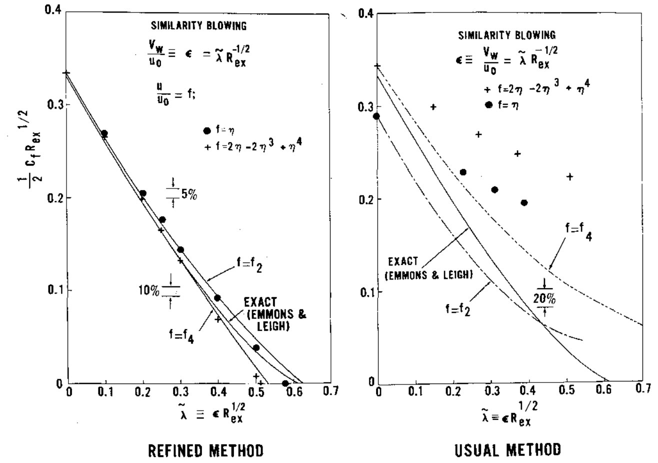

Fig. 1 Skin friction on a porous plate with similarity blowing; f_2 and f_4 are polynomial profiles of 2nd and 4th degrees, respectively, satisfying the compatibility condition (see Zien[3]).

misled MacDonald. This fact is evidenced by the footnote of MacDonald's Table 1, which states clearly that those skin-friction results in the Table, incorrectly quoted as those of Ref. 1†, are obtained by taking the local slope of the assumed velocity profile at the wall. These improper results are apparently based solely on the single equation, i.e., the double-integration equation. It is emphasized here that Volkov's original method,[4] of which Ref. 1 is a direct generalization to allow for surface mass transfer, is to calculate the skin friction by considering the momentum balance across the entire boundary layer, i.e., Eq. (3) of Ref. 1, instead of taking the local slope of the assumed profile at the wall. Of course, the two methods of calculation would lead to the same exact answer if the exact profile were used. It is in this respect that the present method differs from other methods which also exploit the idea of the double integration. The present method is basically a one-parameter type of integral method. However, use is made of two equations, i.e., Eqs. (2) and (3) of Ref. 1, generated by using both the first and the second integration of the original (differential) momentum equation. Only one ordinary differential equation results from this approach, i.e., Eq. (2), which determines δ; C_f then follows immediately from an algebraic equation, Eq. (3). In addition to the standard text books cited by the commentator, the survey paper by Libby et al.[5] is particularly pertinent to the present discussion. The alternate derivation of the double-integration equation given in the Comment is also indicated in this paper.

Secondly, MacDonald's assertion that the improvement of skin-friction calculation due to the present refinement is connected with the satisfaction of the compatibility condition in some modified fashion is of questionable validity. The information contained in Ref. 3, which was also mentioned in a footnote of Ref. 1, indicates plainly that, with the same profiles all satisfying the compatibility condition, the original K-P method still yields results of much poorer quality than the present method. Some typical results are shown in Fig. 1 here which is taken from Zien.[6] The reliability as well as the accuracy of the present method is thus further demonstrated. Mathematically speaking, the imposition of the compatibility condition as a boundary condition to the original boundary-layer equations is obviously unsound. As a matter of fact, previous investigators of integral methods, notably Tani,[7] have shown that the abandonment of such a condition in choosing a profile could sometimes lead to better results.

Thirdly, in regard to the question of the reliability of the present method in treating flows with adverse pressure gradients, the results reported in Ref. 3 should provide a plausible, if not conclusive, answer.

In conclusion, it is the author's opinion that Volkov's procedure combines simplicity and accuracy to an unusual degree, and therefore is worthy of further exploration and extension. It is regretted that MacDonald's comment evidently results from a combination of misinterpretations and misquotations of Volkov's[4] and the author's[1] work.

† The numbers, if properly obtained through the present method, should be 0.333 and 0.336 for $f = \eta$ and $f = 2\eta - 2\eta^3 + \eta^4$, respectively, see Volkov[4] or Zien.[1] Note also that apparently the numbers of m and n associated with "Ref. 1" in the Table are, by mistake, interchanged.

References

[1] Zien, T. F., "A New Integral Calculation of Skin Friction on a Porous Plate," *AIAA Journal*, Vol. 9, No. 7, July 1971, pp. 1423–1425.

[2] Zien, T. F., "Reply by Author to P. S. Granville," *AIAA Journal*, Vol. 10, No. 3, March 1972, p. 367.

[3] Zien, T. F., "Skin Friction on Porous Surfaces Calculated by a Simple Integral Method," *AIAA Journal*, Vol. 10, No. 10, Oct. 1972, pp. 1267–1268.

[4] Volkov, V. N., "A Refinement of the Karman-Pohlhausen Integral Method in Boundary-Layer Theory," *Journal of Engineering Physics*, Vol. 9, No. 5, 1965, pp. 371–374.

[5] Libby, P. A., Morduchow, M. and Bloom, M., "Critical Study of Integral Methods in Compressible Laminar Boundary-Layers," TN 2655, March 1952, NACA.

[6] Zien, T. F., "Study of Boundary-Layer Flows with Pressure Gradient and Mass Transfer by a Simple Integral Method," NOLTR 71–51, March 1971, Naval Ordnance Lab., Silver Spring, Md.

[7] Tani, I., "On the Approximate Solution of the Laminar Boundary-Layer Equations," *Journal of the Aeronautical Sciences*, Vol. 21, July 1954, pp. 487–495 and 504.

Comment on "Calculation of Three-Dimensional Laminar Boundary-Layer Flows"

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IN their recent paper,[1] Fillo and Burbank presented results of computation of three-dimensional, laminar boundary layer for flow past a flat plate with attached cylinder. Comparisons were made between results of exact calculation and that based on the approximating scheme introduced by Wang.[2] It was concluded that good agreement exists in shear stresses and in the prediction of flow reversal for certain regions. However, the approximation as applied failed to predict the flow reversal for flows at a distance of 7.32 cm or greater from the line of symmetry.

It may be pointed out here that Wang's approximating scheme will yield better predictions if a streamline coordinate system (based on the inviscid streamline at the edge of the boundary layer) is used instead of Cartesian coordinates. As discussed in their paper, the term $w u_z$† is important to the flow reversal. Its value, however, differs considerably depending on which coordinate system is used. In Fig. 1, velocities and their components in both Cartesian and streamline coordinates (denoted by subscript s) are shown. Clearly, $u_s \approx u$, $w_s = 0$ for the inviscid flow and $u_s \approx u$, $w_s \ll w$ inside the boundary layer. One can expect, then, that the term $(w u_z)_s$ will also be much smaller than $w u_z$.

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† Symbols follow those of Ref. 1 except where noted.

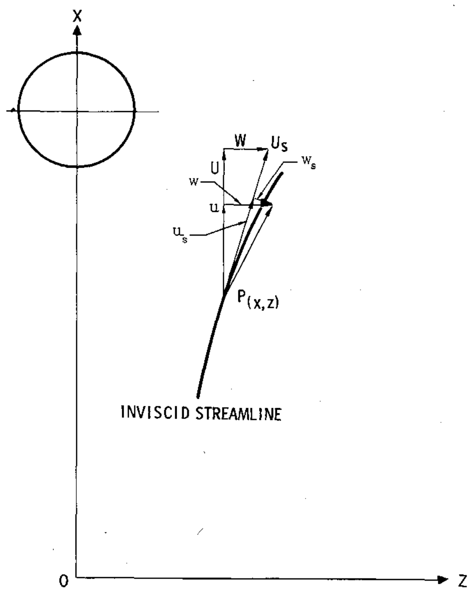

Fig. 1 Velocity components of inviscid and viscous flows at point *P*.

Consequently, its approximation as given in Ref. 2 will be more effective for computation using the streamline coordinate system. This conclusion is further strengthened by the results of Ref. 1. It can be seen that the inviscid velocity component w increases with increasing ζ according to the solution for potential flow. This leads directly to an increase in the viscous velocity component w; which in turn causes the approximation to deteriorate, as shown in the numerical results. On the other hand, use of the streamline coordinate will limit this increase in the value of w, and thereby retain the effectiveness of approximation.

One of the reasons that motivated the present author to select the streamline coordinate system was its better representation of the physical behavior of boundary-layer flow. The comparative study of both exact and approximate methods in Ref. 1 has independently confirmed its effectiveness and applicability.

References

[1] Fillo, J. A. and Burbank, R., "Calculation of Three-Dimensional Laminar Boundary-Layer Flows," *AIAA Journal*, Vol. 10, No. 3, March 1972, pp. 353–355.

[2] Wang, K. K., "An Effective Approximation for Computing the Three-Dimensional Laminar Boundary-Layer Flows," *AIAA Journal*, Vol. 9, No. 8, Aug. 1971, pp. 1649–1651.

Reply by Authors to K. K. Wang

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In the preceding Comment on Ref. 1, Wang emphasizes the role of streamline coordinates in his approximating scheme. Whereas it is true that certain terms, in general, may be 'smaller' computed in a streamline coordinate system compared with Cartesian coordinates, there still remains the question of which terms are important in governing certain flow phenomena such as flow reversal. Near flow reversal inertial terms are important and, consequently, even in streamline coordinates the approximations of Ref. 2 may not be adequate. This observation is what is referred to in Ref. 1 when we said that "Wang's approximation may not be valid in some... particular region of a flowfield."

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Table 1 Approximate three-dimensional boundary-layer calculations of $F_n(0)$

	$\zeta = 3.05$		$\zeta = 6.10$		$\zeta = 7.32$		$\zeta = 9.15$	
ξ-cm	(A)	(B)	(A)	(B)	(A)	(B)	(A)	(B)
0	0.4696	0.4696	0.4696	0.4696	0.4696	0.4696	0.4696	0.4696
0.61	0.4677	0.4677	0.4677	0.4677	0.4678	0.4678	0.4679	0.4679
3.66	0.4631	0.4631	0.4637	0.4637	0.4640	0.4640	0.4646	0.4646
7.93	0.4508	0.4508	0.4529	0.4529	0.4541	0.4541	0.4560	0.4560
10.37	0.4408	0.4408	0.4444	0.4445	0.4464	0.4465	0.4497	0.4497
13.42	0.4231	0.4232	0.4301	0.4302	0.4337	0.4339	0.4398	0.4400
17.69	0.3819	0.3821	0.3990	0.3996	0.4077	0.4084	0.4215	0.4224
20.13	0.3429	0.3433	0.3725	0.3737	0.3869	0.3884	0.4092	0.4109
23.79	0.2398	0.2414	0.3132	0.3179	0.3457	0.3511	0.3920	0.3976
24.40	0.2132	0.2153	0.3002	0.3063	0.3377	0.3445	0.3902	0.3970
25.01	0.1818	0.1848	0.2801	0.2939	0.3295	0.3380	0.3889	0.3972
25.62	0.1442	0.1483	0.2707	0.2809	0.3211	0.3318	0.3885	0.3986
26.23	0.0964	0.1026	0.2541	0.2674	0.3127	0.3263	0.3891	0.4014
26.84			0.2360	0.2535	0.3045	0.3217	0.3910	0.4060
27.45			0.2163	0.2396	0.2967	0.3185	0.3946	0.4128
28.06			0.1948	0.2262	0.2895	0.3172	0.4003	0.4222
28.67			0.1712	0.2138	0.2834	0.3187	0.4086	0.4350
29.28			0.1447	0.2037	0.2789	0.3237	0.4200	0.4517
29.89			0.1142	0.1972	0.2767	0.3335	0.4351	0.4730
30.50			0.0757	0.1962	0.2776	0.3492	0.4549	0.4997
31.11				0.2029	0.2827	0.3724	0.4800	0.5328

Aside from the question of streamline coordinates, the primary deficiency of the approximation in Ref. 1 was its failure to predict flow reversal at a particular distance from the line of symmetry. To see what effect alternative ways of approximating the z or ζ derivatives would have on flow reversal predictions, two additional approximations have been investigated for the problem in Ref. 1

$$u_z \sim U_z = U_\zeta, \qquad w_z \sim W_z = W_\zeta, \qquad g_\zeta = 0 \qquad \text{(A)}$$

The derivatives of velocity are approximated before the transformations

$$F_\zeta = 0, \qquad G_\zeta = 0, \qquad g_\zeta = 0 \qquad \text{local similarity in the } \zeta\text{-direction} \qquad \text{(B)}$$

Tables 1 and 2 are a comparison between approximations (A) and (B) of F_η and G_η at $\eta = 0$.

Again, for small z or ζ values, the approximate calculations predict flow reversal in the u component, the approximate methods predicting flow reversal further downstream from the leading edge than the full three-dimensional calculations. At larger ζ values the full three-dimensional calculations predict flow reversal whereas the approximate methods do not. The same

Table 2 Approximate three-dimensional boundary-layer calculations of $G_n(0)$

	$\zeta = 3.05$		$\zeta = 6.10$		$\zeta = 7.32$		$\zeta = 9.15$	
ξ-cm	(A)	(B)	(A)	(B)	(A)	(B)	(A)	(B)
0	0.4696	0.4696	0.4696	0.4696	0.4696	0.4696	0.4696	0.4696
0.61	0.5307	0.5308	0.5295	0.5297	0.5289	0.5290	0.5277	0.5278
3.66	0.8555	0.8566	0.8477	0.8487	0.8433	0.8442	0.8353	0.8361
7.93	1.351	1.354	1.330	1.333	1.318	1.321	1.297	1.299
10.37	1.664	1.669	1.632	1.636	1.614	1.618	1.582	1.585
13.42	2.094	2.101	2.044	2.049	2.015	2.020	1.965	1.969
17.69	2.798	2.807	2.703	2.711	2.650	2.657	2.559	2.564
20.13	3.278	3.286	3.139	3.146	3.064	3.070	2.935	2.939
23.79	4.172	4.164	3.909	3.906	3.774	3.772	3.552	3.551
24.40	4.351	4.337	4.055	4.048	3.904	3.900	3.660	3.658
25.01	4.543	4.520	4.207	4.195	4.039	4.031	3.770	3.766
25.62	4.750	4.715	4.366	4.348	4.178	4.166	3.881	3.874
26.23	4.973	4.922	4.532	4.507	4.321	4.304	3.993	3.983
26.84			4.706	4.672	4.468	4.445	4.105	4.092
27.45			4.890	4.843	4.620	4.588	4.217	4.200
28.06			5.084	5.021	4.776	4.735	4.329	4.306
28.67			5.290	5.205	4.936	4.882	4.438	4.411
29.28			5.509	5.396	5.100	5.031	4.545	4.511
29.89			5.744	5.593	5.267	5.179	4.648	4.608
30.50			5.992	5.793	5.435	5.326	4.746	4.698
31.11				5.994	5.605	5.467	4.836	4.780